COP 4710, Fall 2009, Homework 3

Problem I.
(a) Proof:
(1) W ->Y (given)
(2) X ->Z (given)
(3) WX ->YX (using IR2 (augmentation) to augment (1) with X)
(4) YX ->Y (using IR1 (reflexivity), knowing that Y subset-of YX)
(5) WX ->Y (using IR3 (transitivity) on (3) and (4))

(b) Proof:
(1) X ->Y (given)
(2) Y ->Z (using IR1 (reflexivity), given that Z subset-of Y)
(3) X ->Z (using IR3 (transitivity) on (1) and (2))

(c) Proof:
(1) X ->Y (given)
(2) X ->W (given)
(3) WY ->Z (given)
(4) X ->XY (using IR2 (augmentation) to augment (1) with X)
(5) XY ->WY (using IR2 (augmentation) to augment (2) with Y)
(6) X ->WY (using IR3 (transitivity) on (4) and (5))
(7) X ->Z (using IR3 (transitivity) on (6) and (3))

(d) Proof:
(1) X ->Y (given)
(2) XY ->Z (given)
(3) X ->XY (using IR2 (augmentation) to augment (1) with X)
(4) X ->Z (using IR3 (transitivity) on (3) and (2))

Problem II. {AC}+ = {A, B, C, D, E}
A,C,F only appear on the left hand side of a FD, so we need to start by computing {ACF}+ to find the key. It happens {ACF}+ contains all attributes of R so ACF is the key. This is the only key.

Problem III. A minimal set of attributes whose closure includes all the attributes in R is a key. Since the closure of {A, B}, {A, B}+ = R, one key of R is {A, B} (in this case, it is the only key).

First, we can test if the schema violates 2NF. This means we want to see if there are non-prime attributes that are functionally dependent on either parts of the key, {A} or {B}, alone. We can calculate the closures {A}+ and {B}+ to determine such attributes:
{A}+ = {A, D, E, I, J}. Hence {A} -> {D, E, I, J} ({A} -> {A} is a trivial dependency)
{B}+ = {B, F, G, H}, hence {B} -> {F, G, H} ({B} -> {B} is a trivial dependency). For both A and B, we can find attributes that depend on them. Therefore, R violates 2NF. It only satisfies 1NF as it does not contain multivalued attributes.

Problem IV. From the question's text, we can infer the following functional dependencies:
{Doctor#, Patient#}→{Diagnosis}
{Treat_code}→{Charge}

Therefore, {Doctor#, Patient#, Date, Treat_code} is the key. Since {charge} is dependent on a subset of the key, this violates 2NF.