Queueing Theory Review Handout

- Arrival rate is \( \lambda \), service rate is \( \mu \), and utilization is \( \rho = \frac{\lambda}{\mu} \)
  - For stability, \( \rho < 1 \) so \( \lambda < \mu \)

- Interested is arrival process, service time distribution, number of servers, system capacity, population size, and service discipline

- Priority can be non-preemptive or pre-emptive (resume and non-resume)

- Little’s Law
  - \( L = \lambda W \)

- Other relations
  - \( L = L_q + \rho \)
  - \( W = W_q + \frac{1}{\mu} \)

- For M/M/1 queue only
  - \( L = \frac{\rho}{(1 - \rho)} \)
  - \( \pi_0 = (1 - \rho) \)
  - \( \pi_k = (1 - \rho)\rho^k \)  \( \{ \) Steady state probabilities \( \} \)

- For M/G/1 (called P-K formula)
  - \( N = \rho + \frac{\rho^2(1 + C_s^2)}{2(1 - \rho)} \)

- Can derive M/M/1 formula from P-K formula (CoV of service distribution is 1)
  - \( N = \rho + \frac{\rho^2(1 + 1)}{2(1 - \rho)} + \frac{\rho^2}{(1 - \rho)} + \frac{\rho^2}{1 - \rho} = \frac{\rho(1 - \rho) + \rho^2}{1 - \rho} = \frac{\rho}{1 - \rho} \)

- Can get M/D/1 formula from P-K formula (CoV of service distribution is 0)
  - \( N = \rho + \frac{\rho^2}{2(1 - \rho)} + \frac{\rho^2}{2(1 - \rho)} = \frac{2\rho(1 - \rho) + \rho^2}{2(1 - \rho)} = \left( \frac{\rho}{1 - \rho} \right) \left( \frac{2 - \rho}{2} \right) \)

- For M/G/m/m (called Erlang-B) – useful for determining number of resources needed
  - \( \Pr[\text{block}] = \frac{(\lambda/\mu)^m}{m!} \sum_{k=0}^{m} \frac{(\lambda/\mu)^k}{k!} \)