Day #8 Exercise – SOLUTIONS

Answer the following questions about queues and simulation a single-server queue.

1) Given an M/G/1 queue with N = 10 customers on average in it and a mean arrival rate of λ = 2 customers per second. What will the mean system delay, D, be? Can we solve this problem? If not, why not?

> Yes, we can solve this problem – we have sufficient information. This problem intentionally used different notation than we have used in class. In this problem N is "our" L and D is "our" W. From Little's Law $L = \lambda W$. We have $L = 10$ and $\lambda = 2$, so then $W = 5$ seconds per customer. The mean system delay, D, is 5 seconds per customer.

2) For the system of (1) what is the mean number of customers in the queueing area, L? What is the mean queueing delay (delay in queueing area), W? Can we solve this problem? If not, why not?

> Again, we have different notation. Here L is "our" Lq and W is "our" Wq. We know that $Lq = L - \rho$ and $Wq = W - (1/\mu)$. But, we don't know (and cannot derive) ρ and μ from (1), so we can't solve the problem.

3) What is the ratio for the mean number of customers in the system (L) for M/M/1 compared to M/D/1 as a function of ρ?

> See next page for the derivation.

4) Here below is the main program of the simple M/M/1 simulation. Modify the code to model an M/D/1/10 queue. Also, add a check for stability.

> Text in arial bold is new. Note also the strike-outs.

```
<SNIP SNIP>

  // Main simulation loop
  assert(Ta > Ts);
  while (time < end_time)
  {
    if (t1 < t2)                      //** Event #1 (arrival)
    {
      time = t1;                      // Set time to that of current event
      if (n < 10)                     // If room left in system
        n++;                          //  increment number of customers in system
      t1 = time + exponential(Ta);    // Assign time for the next arrival event
      if (n == 1)                     // If first customer in system then
        t2 = time + ~~exponential(~~Ts~~)~~; //   assign its departure time
    }
    else                               // *** Event #2 (departure)
    {
      time = t2;                       // Set time to that of current event
      n--;                             // Decrement number of customers in system
      if (n > 0)                       // If customers in system then
        t2 = time + ~~exponential(~~Ts~~)~~;  //   assign next departure time
      else                             // If no customers in system then
        t2 = end_time;                 //   assign next departure to "infinity"
    }
  }

<SNIP SNIP>
```

Solutions to (3):

For M/M/1 we know that $N = \rho/(1-\rho)$. We need to derive N (as a function or ρ) for M/D/1. We do this from the P-K formula:

$$N = \rho + \frac{\rho^2\left(1+C_s^2\right)}{2(1-\rho)}$$

We know that $C_s^2 = 0$ for M/D/1. So,

$$N = \rho + \frac{\rho^2}{2(1-\rho)} = \frac{2\rho(1-\rho)}{2(1-\rho)} + \frac{\rho^2}{2(1-\rho)} = \frac{2\rho+\rho^2}{2(1-\rho)} = \left(\frac{\rho}{1-\rho}\right)\left(\frac{2-\rho}{2}\right)$$

So, then is it obvious that $N_{MM1}/N_{MD1} = 2/(2-\rho)$. As ρ approaches one, the ratio approaches 2.