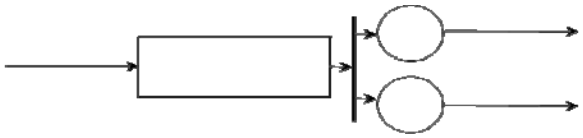
Day #7 Exercise – SOLUTIONS

Answer the following questions about queues and queueing:

1) What is λ? What is μ? What is ρ? How is ρ determined from λ and μ? When is utilization and offered load the same? When is it not the same?

> λ is the arrival rate of customers. μ is the service rate of customers. ρ is the utilization and is computed as $\rho = \lambda/\mu$. Utilization and offered load are the same when there is no loss in the system. If there is loss in the system then offered load can be higher (but never lower) than utilization.

2) Sketch an M/D/2/10 queue



Capacity = 8
Arrivals are Poisson, service is deterministic

3) Which will have lower delay – N M/M/1 queues or one M/M/N queue (all servers have the same service rate) Why?

> The M/M/N will have lower delay because a server will never be idle when there are customers to be servered. In an N x M/M/1 system there may be idle servers when other servers have customers queued.

4) Given a queueing system with on average 10 customers in it and a mean arrival rate of 2 customer per second, what will the mean wait be?

> This is a Little's Law problem. $L = \lambda W$. We have $L = 10$ and $\lambda = 2$, so then $W = 5$ seconds per customer.

5) Given an M/M/1 system with mean service time of 3 seconds per customer and mean arrival rate of 2 customers per second, what is the mean number of customers, *L*, in the system? What is the formula for *L* for an M/M/1 system?

> The mean service rate (μ = 1/3 customers per second) is less than the service rate, so the queue is unstable. If this were not the case, we would use $L = \rho / (1 - \rho)$ where $\rho = \lambda/\mu$ to solve the problem.

6) Given an M/M/1 system with mean arrival rate of 40 customers per second and mean service rate of 60 customers per second, what is the mean wait, *W*, in the system? In the queue only (that is *Wq*)?

> We can use $L = \rho / (1 - \rho)$ to solve for L and then Little's Law to get $W = L / \lambda$ and then $Wq = W - (1/\mu)$. We have $\rho = 40/60 = 2/3$. Then, $L = 2$ customers and $W = 1/20$ sec and $Wq = 3/60 - 1/60 = 1/30$ sec. Always give the units for a number (that is not unitless as is ρ).