Day #4 Exercise – SOLUTION

Solve/answer the following:

a) What is a random variable?

A random variable is a function that maps a real number to every possible outcome in the sample space.

b) What is the difference between a combination and a permutation? Be precise.

Both combinations and permutations are sampling of R items without replacement from N items. In a permutation the order of the sampled items matters (so, A-B-C is different from A-C-B), in a combination it does not (so, A-B-C is the same as A-C-B). A combination is a set. Thus, there are R! less combinations than permutations.

c) What is the mean and standard deviation for a uniform continuous distribution with minimum value 5 and maximum value 15?

Mean = (5 + 15) / 2 = 10

Variance = $(15 – 5)^2/12 = 100/12$. Standard deviation is sqrt(100/12) = 2.887

d) Repeat (a) for a uniform discrete distribution.

Mean = (5 + 15) / 2 = 10

Variance = $((15 – 5 + 1)^2 – 1)/12 = 120/12$. Standard dev is sqrt(120/12) = 3.162

e) What is the mean and standard deviation for an exponential distribution with mean rate (λ) of 10.0?

Mean = $1/\lambda = 0.1$

Variance = $(1/\lambda)^2 = 0.01$, Standard deviation is then sqrt(0.01) = 0.1

f) Assume that the probability of a server failing over night is $p = 0.1$. If you have 10 servers, what is the probability that 2 servers will have failed over night?

This is a binomial distribution where success is failing overnight.

Combin(10,2)*$(0.1)^2$*$(1-0.1)^{10-2}$ = 0.194

g) Assume that the probability of a server failing over night is $p = 0.1$. If you have 1000 servers, what is the probability that 100 servers will have failed over night?

Combin(1000,100)*$(0.1)^{100}$*$(1-0.1)^{1000-100}$ = 0.042

Better might be to approximate this with a Poisson distribution (λ = 0.1 * 1000 = 100)

$$\frac{\lambda^{k}}{k!}\cdot e^{-\lambda} = 0.039861$$

h) Given a Poisson process with mean rate 20 arrivals per second, what is the probability of having 15 arrivals in any one second?

$$\frac{\lambda^{k}}{k!}\cdot e^{-\lambda} = 0.051649$$